* The Quotient Topology :-

Definition: (Quotient map) :-

→ Let X and Y be topological spaces.
Let $p : X \longrightarrow Y$ be surjective map.
The map p is said to be a quotient map.
Provided a subset V of Y is open in Y
Iff $p^{-1}(V)$ is open in X.

* This condition is stronger than continuity
So, it is called stronger continuity.

→ An equivalent condition is to require
that a subset A of Y be closed in Y.
iff $p^{-1}(A)$ is closed in X.

$$f^{-1}(Y - A) = X - f^{-1}(A).$$

#

* Another way of describing a quotient map as :-

Def:- A subset C of X is said to be
saturated (w.r.t the surjective map $p : X \to Y$)
if C contains every set $p^{-1}(\{y\})$ that it meets.

Thus C is saturated if it equals the
Complete inverse image of a subset of Y.

To say that p is a quotient map
iff p is continuous and p maps
saturated open (closed) sets of X to
open (closed) sets of Y.

* Two special kinds of quotient maps are
the open maps and the closed maps.

Def^n: (1) A map $f : X \longrightarrow Y$ is said to be an
open map. If for each open set U of X,
the set $f(U)$ is open in Y.